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Dynamics of $O(N)$ chiral supersymmetry at finite energy density

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Abstract

We consider an $O(N)$ version of a massive, interacting, chiral supersymmetry model solved exactly in the large N limit. We demonstrate that the system approaches a stable attractor at high energy densities, corresponding to a non-perturbative state for which the relevant field quanta are massless. The state is one of spontaneously broken $O(N)$, which, due to the influence of supersymmetry, does not become restored at high energies. Introducing soft supersymmetry breaking to the Lagrangian results in scalar masses at the soft breaking scale m_s independent of the mass scale of supersymmetry μ , with even smaller masses for the fermions.

1 Introduction

Supersymmetry[1, 2] at finite temperature has a number of interesting properties[3, 4, 5, 6, 7]. Foremost is the appearance of a massless fermion mode, the so-called Goldstino. As finite temperature corrections are different for particles obeying Bose-Einstein and Fermi-Dirac statistics, a finite temperature state is not invariant under supersymmetry transformations. Since the transformation parameter of supersymmetry is a Grassmann variable, the breaking of supersymmetry therefore implies the existence of a massless Goldstone fermion.

One situation where such properties might be particularly relevant is during the reheating stage after inflation. It has been shown that light fermions may play an important role in the reheating process[8], and so it is important to know whether massless or nearly massless fermions might be a requirement in supersymmetric models due to the phenomenon of the Goldstone fermion. However, there is an important distinction between finite temperature physics and the far from equilibrium, finite energy density situation relevant to inflation.

Despite the rich activity in out of equilibrium field theory, the case of supersymmetric models has not yet been properly investigated. In this letter, we address a supersymmetric model including all of the superpartner degrees of freedom, including the fermions. The model is one with a global $O(N)$ symmetry which we solve exactly in the limit of large N [9].

Working at large finite energy density, but far from thermal equilibrium, we study the dynamics of the model and see that the system indeed evolves to an attractor state for which a set of fermion modes becomes massless. We argue that this state is related to the spontaneous breaking of the $O(N)$ symmetry and that, therefore, the fermions are massless because they are superpartners of Goldstone bosons. Somewhat surprisingly, the $O(N)$ symmetry is not restored at high energy as would normally be expected[10, 11]. As the energy is increased, the contributions from the various superpartners to the effective masses of the particle modes cancel each other, a result due to supersymmetry.

We stress that although there are $O(N)$ symmetric vacua in this theory which are degenerate in energy with the $O(N)$ broken vacua, at high energies the system invariably evolves to the $O(N)$ broken vacua for which there are massless fermions. In this way, the final vacuum state of the system is predetermined by the high energy evolution. The possibility that there may be selection rules between degenerate vacua resulting from the early evolution of the universe is a primary result of this work.

The model we study contains an $O(N)$ singlet superfield, characterized by the non-zero vacuum expectation value of the scalar component, $\phi(t)$, and an $O(N)$ vector supermultiplet. The $O(N)$ vector fields have time dependent masses determined by an order parameter $m^2(t)$ reflecting the internal dynamics of the system. If $\phi(t = 0)$ is far enough from the supersymmetric vacua, i.e., if the initial energy density is sufficiently large, the system evolves in such a way that

the order parameter $m_-^2(t)$ asymptotically vanishes, corresponding to zero masses for all of the fields of the $O(N)$ vector multiplet. The state is indicative of the spontaneous breaking of the $O(N)$ symmetry.

We note that while the ground state upon which this finite energy state is built is supersymmetric, the energy is distributed differently among the fermions and bosons due to their differing statistics. This is analogous to what is known from equilibrium studies at finite temperature[3, 4, 5, 6, 7]. In a cosmological context it is expected that the quanta in the highly excited state would become diluted with expansion and the universe would eventually find itself approaching the underlying ground state.

It is also worth mentioning that the state is stable to the introduction of small soft supersymmetry breaking to the Lagrangian. Such terms explicitly break the supersymmetry, while not introducing any terms which disrupt the supersymmetric solution to the hierarchy problem. Here we find that the qualitative behavior is unchanged by such terms as the system still evolves to an attractor $O(N)$ broken state. Furthermore, such terms in fact introduce small masses to one set of scalar fields of order the soft breaking scale m_s , which is taken to be much smaller than the overall scale of supersymmetry μ . The fermions meanwhile gain a mass several orders of magnitude smaller.

Although this is only a toy model, the important features of the model – the combination of continuous symmetries leading to Goldstone bosons and supersymmetries leading to massless superpartners – are completely general. We also note that in models without additional continuous symmetries, such as the ordinary Wess-Zumino model, the far from equilibrium system is found to evolve toward a state for which the fermions become massless; this case will be studied in detail in a future publication[12]. The results may be important to our understanding of aspects of cosmology, such as supersymmetry based inflation models[13] and electroweak baryogenesis[14, 15, 16, 17].

2 The model

The $O(N)$ extension of the Wess-Zumino model[18] consists of a chiral superfield multiplet $S_0 = (A_0, B_0; \psi_0; F_0, G_0)$, which acts as a singlet under $O(N)$, coupled to N chiral superfields $S_i = (A_i, B_i; \psi_i; F_i, G_i)$ with $i = 1 \dots N$, transforming as a vector under $O(N)$. Here, A and F are real scalars, B and G are pseudo-scalars, and ψ is a Majorana fermion. The superpotential has the form

$$\begin{aligned} W(S_0, S_i) &= \frac{1}{2} M S_0^2 + \frac{\kappa}{6\sqrt{N}} S_0^3 \\ &+ \sum_{i=1}^N \frac{1}{2} \mu S_i^2 + \frac{\lambda}{2\sqrt{N}} S_0 S_i^2 . \end{aligned} \quad (1)$$

We expand in terms of the component fields and eliminate the auxiliary fields

via their equations of motion. To allow for a proper large N limit, the expectation value of A_0 must be of order \sqrt{N} . We therefore set

$$\langle A_0 \rangle = \sqrt{N}\phi, \quad \langle B_0 \rangle = 0. \quad (2)$$

The field B_0 is taken to have zero expectation value for the sake of simplicity. We assume that the initial state satisfies the $O(N)$ symmetry which requires that $\langle A_i \rangle = \langle B_i \rangle = 0$. It is convenient to take full advantage of this symmetry to define fields A , B , and ψ such that $\sum_i A_i A_i = N A^2$, $\sum_i B_i B_i = N B^2$, and $\sum_i \bar{\psi}_i \psi_i = N \bar{\psi} \psi$. The resulting Lagrangian to leading order in N is

$$\begin{aligned} \frac{\mathcal{L}}{N} = & \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} \partial_\mu A \partial^\mu A + \frac{1}{2} \partial_\mu B \partial^\mu B \\ & - \frac{1}{2} \phi^2 \left(M + \frac{1}{2} \kappa \phi \right)^2 - \frac{1}{2} (\mu + \lambda \phi)^2 (A^2 + B^2) \\ & - \frac{\lambda}{2} \left[M\phi + \frac{1}{2} \kappa \phi^2 + \frac{1}{4} \lambda (A^2 - B^2) \right] (A^2 - B^2) \\ & + \frac{i}{2} \bar{\psi} \gamma^\mu \partial_\mu \psi - \frac{1}{2} \mu \bar{\psi} \psi - \frac{1}{2} \lambda \phi \bar{\psi} \psi. \end{aligned} \quad (3)$$

The system may be completely characterized by the equation of motion for the mean field ϕ

$$\ddot{\phi} + \frac{M + \kappa \phi}{\lambda} m_-^2 + \lambda m_\psi \langle A^2 + B^2 \rangle + \frac{1}{2} \lambda \langle \bar{\psi} \psi \rangle = 0, \quad (4)$$

and by the time dependent masses of the ψ , A , and B fields:

$$m_\psi = \mu + \lambda \phi, \quad (5)$$

$$m_A^2 = m_\psi^2 + m_-^2, \quad (6)$$

$$m_B^2 = m_\psi^2 - m_-^2, \quad (7)$$

with,

$$m_-^2 \equiv \lambda \left[M\phi + \frac{1}{2} \kappa \phi^2 + \frac{1}{2} \lambda \langle A^2 - B^2 \rangle \right]. \quad (8)$$

The expectation values, $\langle \bar{\psi} \psi \rangle$, $\langle A^2 \rangle$, and $\langle B^2 \rangle$ are determined in the usual way in terms of the non-equilibrium mode functions for the individual fields. General details, including the renormalization procedure may be found in Refs. [8, 19, 20]. Specific details for this model will be provided in future work[12].

Note that the appearance of $\langle A^2 - B^2 \rangle$ on the right hand side of the expression for m_-^2 , Eq. (8), means that this expression plays the role of a gap equation which must be satisfied by the dynamics. Also note that the sum rule $m_A^2 + m_B^2 - 2m_\psi^2 = 0$ is automatically satisfied for all times as required by supersymmetry.

Through use of the equations of motion, it is straightforward to show that the variation of the Lagrangian (3) vanishes under supersymmetry transformations up

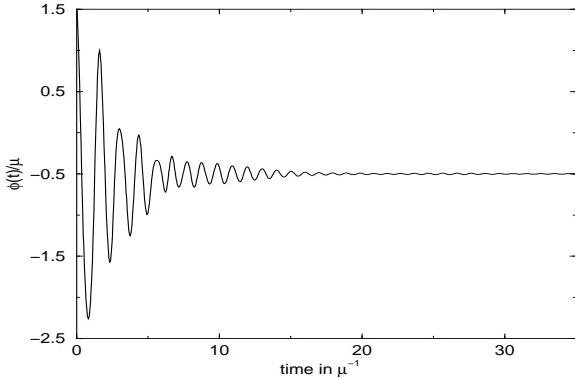


Figure 1: Zero mode evolution; the parameters are $\mu = 1$, $M = 4$, $\kappa = 1$, $\lambda = 2$, and $\phi(0) = 1.5$. All masses are scaled by μ which is arbitrary.

to a total derivative. To this order, the Lagrangian is completely supersymmetric with ϕ acting as a classical background field.

We stress that the evolution determined by equation (4) and the time dependent masses (5) – (8) exactly solves the quantum field theoretical system described by the superpotential (1) in the $N \rightarrow \infty$ limit to all orders in perturbation theory. We have made no further approximations and the only simplifications, eg., $\langle B_0 \rangle = \langle A_i \rangle = \langle B_i \rangle = 0$, correspond to choices of initial conditions.

3 Numerical results

Finite energy density is imposed via out of equilibrium initial conditions for the zero mode ϕ . The evolution toward the non-perturbative attractor state is depicted in Fig. 1 with the corresponding masses of the fields in Fig. 2. We note the following characteristics. The evolution begins with large oscillations of ϕ over the entire classically allowed range of evolution. During this initial period, the field fluctuations $\langle A^2 \rangle$ and $\langle B^2 \rangle$ grow. After a relatively short period of time, the mean field settles down precisely to the point $\phi = -\mu/\lambda$. The result is that the N fermions become massless.

Interestingly, the scalar fluctuations continue to grow until a state is formed for which the fields A and B are massless as well (Fig. 2). This configuration remains completely stable. We also find that this behavior persists up to energy densities much higher than any natural scale in the problem, indicating that there is no symmetry restoration and that this represents the generic behavior of the system. This is made possible by the cancellation of contributions from $\langle A^2 \rangle$ and $\langle B^2 \rangle$ in the gap equation (8), a direct consequence of supersymmetry, which allows these field fluctuations to become arbitrarily large while not changing their contributions to the masses of the field quanta. The growth of these fluctuations also provides the mechanism for driving the fermion mass to zero, see Eq. (4).

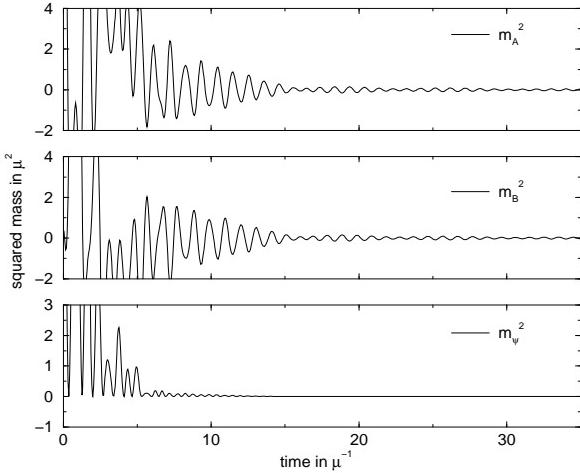


Figure 2: The effective field masses squared, m_A^2 (top), m_B^2 (middle), and m_ψ^2 (bottom) with the same parameters as in Fig. 1. Each effective mass vanishes at late times.

In order to understand the supersymmetric field configuration reached by the non-equilibrium time evolution, it is illuminating to study the effective potential for static field configurations as a function ϕ . It is obtained (see, e.g. [10]) by maximizing with respect to m_-^2 the potential

$$\begin{aligned} V(\phi, m_-^2) = & \left(M\phi + \frac{1}{2}\kappa\phi^2 \right) \frac{m_-^2}{\lambda} - \frac{m_-^4}{2\lambda^2} \\ & + \frac{1}{64\pi^2} \left[g(m_A^2) + g(m_B^2) - 2g(m_\psi^2) \right], \end{aligned} \quad (9)$$

where $g(m_i^2) = m_i^4(\ln(m_i^2/m^2) - 3/2)$.

The effective potential $V(\phi)$ is plotted in Fig. 3 along with the tree level potential. We see that the effective potential has a minimum at $\phi = -\mu/\lambda$, and one finds that at this point all the masses vanish. To understand this new minimum, it is helpful to remember that, if one allows for states which break $O(N)$, such that, for example, $\langle A_1 \rangle = \sigma \neq 0$ then one finds $O(N)$ breaking minima at $\lambda\phi = -\mu$ and $\lambda\sigma = \pm\sqrt{2\mu M - \kappa\mu^3/\lambda}$. As a result of the convexity theorem, the exact large N effective potential must be flat between these minima as in a Maxwell construction. Hence, there must be a new minimum in the full effective potential as shown in the figure.

The state that is reached by the evolution is therefore a phase consisting of different spontaneously broken $O(N)$ states in coexistence.

The values of ϕ and the mass parameters obtained at late times in the out of equilibrium evolution correspond precisely to such a state. The state itself is a highly excited one and is time dependent in a coherent way. Furthermore, we find that the system is attracted towards this configuration once the energy density

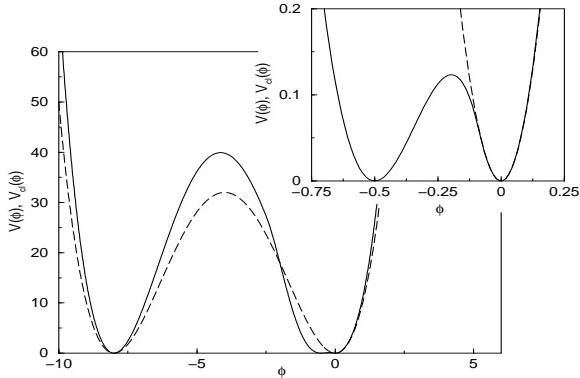


Figure 3: The large N effective potential (solid line), showing the $O(N)$ symmetric ground states at $\phi = 0$ and $\phi = -2M/\kappa = -8$ and the spontaneously broken $O(N)$ coexistence phase at $\phi = -\mu/\lambda = -1/2$. The inset frame magnifies the region around the $O(N)$ broken minimum. In each case, the tree level potential (dashed line) is shown for comparison. The parameters are as in Fig. 1.

is sufficiently high.

As a side note, we point out that the effective potential in the ϕ direction is non-convex. This has to do with the fact that ϕ acts as a simple classical zero mode whose fluctuations are completely negligible in the leading order large N limit.

Next, we introduce soft symmetry breaking to the $O(N)$ model via a scalar mass m_s for the A and B fields such that $m_A^2 = m_\psi^2 + m_-^2 + m_s^2$ and $m_B^2 = m_\psi^2 - m_-^2 + m_s^2$. Fig. 4 shows the result for a value $m_s^2/\mu^2 = 10^{-4}$. We see again that the system reaches the attractor state, and the explicit soft symmetry breaking terms provide a mass for the A field equal to $\sqrt{2}m_s$, while the B field remains massless. The fermion mass is found to be three orders of magnitude smaller.

We make the following conclusions. First, supersymmetry may play a very important role in the dynamics of the early universe beyond ordinary model building. The requirement of massless fermions appearing in the spectrum, in particular, may be important to inflationary and reheating dynamics and could also play a significant role in baryogenesis. Furthermore, the constraints of supersymmetry can lead to a preferential choice between multiple degenerate vacua through the existence of attractor states at finite energy density. Such issues are deserving of further study.

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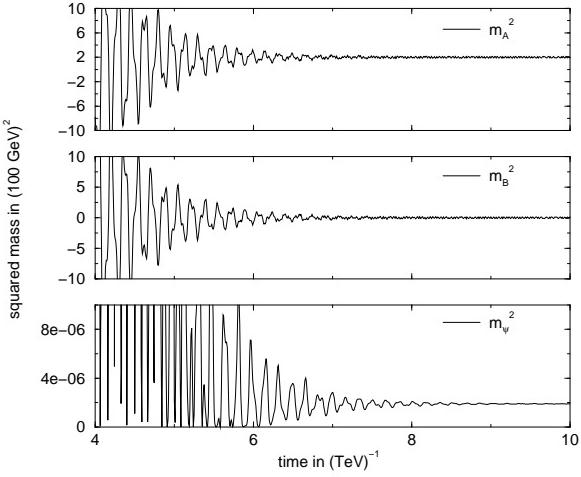


Figure 4: The effective field masses squared $m_A^2(t)$, $m_B^2(t)$, and $m_\psi^2(t)$ in the $O(N)$ broken phase including soft masses for the A and B . The parameters are $\mu = 10$ TeV, $M = 40$ TeV, $m_s = 100$ GeV, $\kappa = 1$, $\lambda = 2$, $\phi(0) = 15$ TeV. The late time values correspond to $m_A = \sqrt{2}m_s = 140$ GeV, $m_B = 0$, and $m_\psi = 140$ MeV.

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